PROCESSING OF TEXTURED SURFACES REPRESENTED AS SURFEL SETS: REPRESENTATION, COMPRESSION AND GEODESIC PATHS

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ABSTRACT

A method for representation and lossy compression of textured surfaces is presented. The input surfaces are represented by surfels (surface elements), i.e., by a set of colored, oriented, and sized disks. The position and texture of each surfel are mapped onto a sphere. The mapping is optimized for preservation of geodesic distances. The components of the resulting spherical vector-valued function are decorrelated by the Karhunen-Loève transform and represented by spherical wavelets. Successful representation and reconstruction is demonstrated. Methods for geodesic distance computation on surfaces represented by surfels are presented.

1. INTRODUCTION

Various modern image processing applications are inherently three dimensional. Textured three dimensional models are used in video games, virtual shops, computer-aided design and medical image databases. This research is concerned with the processing of textured 3D surfaces, in particular with their representation and compression.

Textured 3D models can be either synthetic or real. Synthetic models are generated by computer graphics; real models are obtained by 3D scanning techniques. Real time transmission and compact storage of detailed 3D models require compression of textured 3D models. Efficient indexing in large 3D image databases call for convenient model representations.

Three dimensional models appear in various formats. Triangulated meshes are common in computer graphics; voxel representations are dominant in medical imaging. Recently, Pfister *et al* [12] developed a surface representation
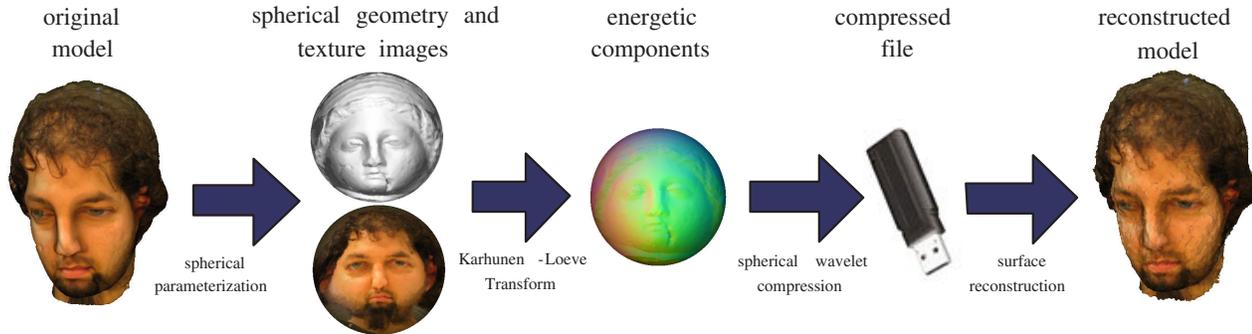
scheme based on *surfels* (surface elements). A surfel S_i is a disk with center location P_i, normal vector $\vec{n}_i$, color 3-tuple (r_i, g_i, b_i), and radius r_i. In this representation, a surfel can be viewed as an extended point sample, without any connectivity implied. Surface representation by surfel sets is gaining popularity, and is a natural output format for important classes of 3D scanners (laser and structured light, shape from shading and texture, photometric stereo, tactile). This research focuses on the compression and representation of textured surfaces represented by surfel sets.

A compression scheme for surfel-based surfaces was recently presented by Ochotta and Saupe [11]. Their method consists of dividing the surfaces into patches and compression of the patch height field above a suitable reference plane. Our approach to surface compression and representation is different in several respects. In particular, in many typical applications the surface is closed and of genus 0. In these cases we may use a single joint (spherical) parameterization in place of a collection of patches. Indexing in 3D surface databases, and invariance to geometric transformations, are easier to accomplish when the surface is represented as a whole.

The proposed method for representation and compression of textured surfel-based surfaces is described in section 2. Geodesic distances play an important role in the suggested representation. The measurement of geodesic distances on surfel-based surfaces is considered in section 3.

2. REPRESENTATION AND COMPRESSION OF TEXTURED SURFACES

The compression of surfaces that are represented by triangle meshes has received significant attention, see e.g. [18, 8, 2]. In some cases, the preservation of the mesh connectivity is an important consideration [18]. In others, remeshing takes place as part of the compression and decompression cycle [2]. In surfel-based surface representation there is no mesh, hence connectivity need not be preserved. Neverthe-

original model   spherical geometry and texture images   energetic components   compressed file   reconstructed model

spherical parameterization   Karhunen -Loeve Transform   spherical wavelet compression   surface reconstruction

Fig. 1. The proposed method for representation and compression of textured 3D surfaces.

less, in analogy to the remeshing notion, the surface can be "resurfeled" as part of the process. We focus on lossy compression, that maintains good visual quality at high compression ratios.

In the suggested method for representation and compression of textured surfel-based surfaces, the underlying continuous surface is optimally mapped onto the unit sphere. The mapping of each surfel includes its spatial coordinates and color components as a vector. On the sphere, the mapped vectors are regarded as samples of a continuous vector valued (multilayered) function. The correlation between the layers is removed, and layer-wise energy compaction is optimized, using the Karhunen-Loève transform. The resulting spherical vector valued function is represented by spherical wavelets [14, 17]. The wavelet coefficients are quantized and entropy encoded for transmission or storage. The proposed algorithm is illustrated in Fig. 1. The rest of this section outlines the key steps of spherical parameterization, Karhunen-Loève transformation and spherical wavelet representation.

Spherical parameterization is the mapping of points from the surface to the sphere. Praun & Hoppe [13] mapped vertices of a triangulation sequentially, minimizing a local stretch measure. Global relaxation steps at preset intervals complement the process. This method was later used as part of a compression scheme for surface meshes [6]. Zwicker & Gotsman [20] consider spherical parameterization of a point set by constructing a k-nearest neighbor graph and iteratively reducing the corresponding Laplace-Beltrami operator on the sphere. We use a globally optimal algorithm, that was originally developed by Cox & Cox [1], and later extended and applied to surface analysis by Elad & Kimmel [3]. A stress functional quantifies the discrepancy between geodesic distances between a set of points on the surface and the corresponding distances on the sphere. The functional is minimized using the conjugate gradients algorithm. To reduce storage requirements ($O(N^2)$ in the number of mapped points) and run-time, only a subset of the surfels are mapped by stress minimization, and the mapping

of the rest is obtained by interpolation [4].

Following their mapping onto the sphere, the position and texture functions are resampled [4] on a spherical grid, at roughly the original resolution. We use a grid based on an icosahedron. The spherical triangles corresponding to the icosahedron faces are recursively divided to create the spherical grid.

The mapping of the surface onto the sphere results in a vector at each point on the spherical grid. The components (layers) are the spatial coordinates and the color components of the texture at a point. The Karhunen Loève transform is used to decouple the layers and optimize layerwise energy compaction. Each layer is then represented by spherical wavelet decomposition via the lifting scheme of Sweldens [17], adapted for spherical functions by Schröder & Sweldens [14]. In the lifting scheme, the wavelet basis function is not formulated explicitly, but is implicitly 'lifted' from level to level, using a 'mother' wavelet function. A lifting stage creates a down-scaled version of the original function, and wavelet coefficients for reconstruction. In this paper the butterfly mother function was used. The resulting wavelet coefficients are coded using the zerotrees method [15].

In the reconstruction stage, a surfel is created for each point on the spherical grid. Its position and color are known from the code; its orientation and radius are derived from the neighborhood.

3. COMPUTING GEODESIC PATHS ON SURFEL-BASED SURFACES

Geodesics for point clouds have been considered before. In Memoli & Sapiro [10] geodesic curves are constrained to regular grid points on an ϵ-neighborhood of the associated surface. In Hofer & Pottmann [5] curves are constrained to be in the corresponding moving-least-squares surface. The surfel-based representation is different from plain point clouds, because surfels are oriented and have a
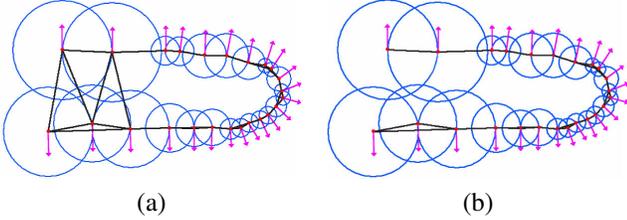
Fig. 2. (a) Canonical SIG, and (b) SIG extended with surfel orientations.

size attribute.

The geodesic distance between two arbitrary points P_1 and P_2 on a surface is defined as the length of the shortest path between them on the surface. As discussed above, estimates of geodesic distances are needed in order to reduce the parameterization distortion. To compute geodesics on a specific surface, its connectivity must be known. We construct a weighted graph to describe surfel connectivity. Among the various graphs proposed in the literature [7] for point samples, we chose to adapt the spheres-of-influence graph (SIG) [9]. It accommodates variations in the point sample density and produces a good description of non-smooth surfaces [7, 9].

The sphere centered at point sample (surfel position) v_i with radius given by the distance to the nearest neighbor is called the sphere of influence of v_i. The SIG is the graph with vertices v_i in which two vertices v_i and v_j are connected by an edge e_{ij} if the corresponding spheres of influence intersect. Each edge e_{ij} is weighted by the Euclidean distance between the connected vertices. We extend the concept of SIG by considering surfel orientations, i.e., their normal vectors. We connect two vertices by an edge only if the dihedral angle between their surfel normal vectors is less than a certain threshold angle θ_t.

As shown in [9], in case of noisy or irregularly sampled point clouds, a SIG could produce small clusters of connected points that are inter-cluster disconnected even though they are part of the same surface. To overcome this problem, different approaches that focus on modifying the radii of the spheres of influence can be applied. In [9] the radius is set to the distance to the k-th nearest neighbor with $k > 1$, a parameter of the method. This may produce extraneous long edges in the graph requiring an additional edge pruning step. In our case of surfel-based models we may simply set r_i to be the radius of the surfel S_i. In fact, the radius of a surfel disk identifies the contribution of its sample point in representing the underlying surface, thus two surfels will be connected by an edge only if their areas of contribution to the approximated surface intersect. In addition, we require that two surfels connected by an edge have approximately the same orientation. Using this approach undesirable edges will be discarded, see Fig. 2.
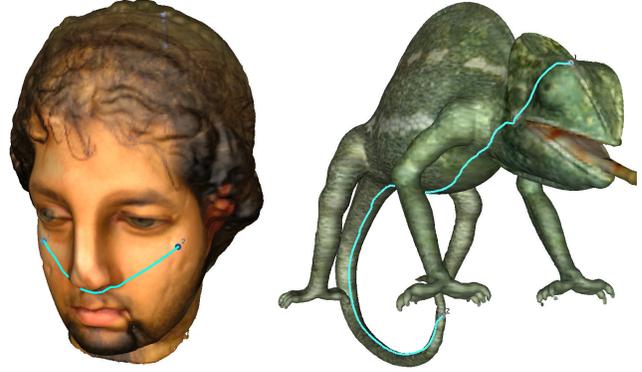
Fig. 3. Geodesic paths (in cyan) on two surfel based models.

The geodesic distance between surfels, that is needed in the spherical parameterization step of the proposed representation scheme, can be computed using Dijkstra's algorithm. If needed, the geodesic path can also be constructed: Fig. 3 shows examples of the geodesic path between selected points on surfel-based surfaces.

Our approach provides fast approximation of the geodesic distance on surfel-based surfaces. The computation of the graph takes $O(|V|\log|V|)$ time [7], where V is the set of vertices. The complexity of computing the geodesic distance between a given surfel and all other surfels in the model using Dijkstra's algorithm is $O(|E| + |V|\log|V|)$, where E is the set of edges [16]. This procedure must be executed M times, where M is the number of control points.

4. RESULTS AND DISCUSSION

Fig. 4a is the Igea (Venus) 3D model [19], consisting of 134345 surfels (120 bits/surfel), mapped with a texture of another head. Compression to 0.89 bits/surfel yields the reconstruction shown in Fig. 4b. Allowing 3.56 bits/surfel leads to better reconstruction, Fig. 4c. To allow evaluation of the geometry compression, Fig. 4d shows the outcome of compression without texture, to the accuracy of (c). Since texture is not included in the data to be compressed, the geometric reconstruction (d) is obtained with 1.39 bits/surfel.

We presented a compression scheme for textured surfaces represented as surfel sets. The spherical parameterization of the surface implies that the topology of the surface to be compressed should be compatible with a sphere. An important advantage of the spherical parameterization is the straightforward extraction of shape and texture features that are invariant to translation, rotation and scale. Invariant features are useful for indexing in 3D model databases.

Our approach for computing geodesics based on neighborhood graphs was motivated by the application of spherical parameterization, in which it is not crucial to have exact
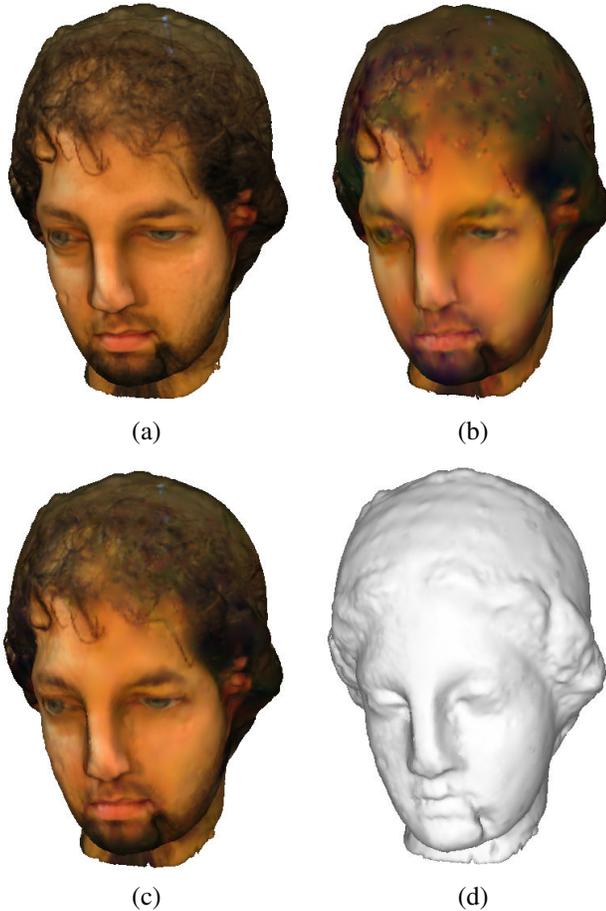
Fig. 4. (a) 3D model with texture, 134345 surfels, 120 bits/surfel. (b) Following compression to 0.89 bits/surfel. (c) Following compression to 3.56 bits/surfel. (d) Following compression without texture, to the accuracy of (c), 1.39 bits/surfel.

geodesic distances. Our method provides a good balance between performance and efficiency. For higher precision one may introduce a relaxation algorithm that improves the piecewise linear geodesic approximation. For this purpose we propose to apply an energy minimization taking into account euclidean distance, closeness to the surface, and distribution of the sampling of the geodesics.

Further research problems that we are addressing are to define a distortion measure that combines geometric reconstruction quality with texture fidelity, and to derive an algorithm to assign an optimal bit allocation between point positions and texture.

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